Compressor Design Method in the Supercritical CO$_2$ Applications

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ABSTRACT

The supercritical CO$_2$ Brayton cycle has been recently attracting more attention compared to other common energy conversion cycles, chiefly due to its higher thermal efficiency with the relatively low temperature at the turbine inlet compared to its conventional counterparts. Centrifugal compressor operating conditions in the supercritical Brayton cycle are preferably located near the critical point to get advantage of low compressibility factor and eventually low compressor work. In this paper, the design of the compressor using the enthalpy loss models in the supercritical CO$_2$ region is investigated and the accuracy of the loss models near the critical using real gas is validated. Due to high density of the working fluid near the critical point, compressor size is relatively small. It is noticed that, the friction loss plays a significant role among all loss sources. Therefore, more attention is paid on the skin friction loss and friction coefficient estimations. Results are compared to the experimental measurements conducted at Sandia National Laboratories.

INTRODUCTION

Recently, there has been significant growing interest in the supercritical CO$_2$ (sCO$_2$) Brayton cycle as an alternative power conversion cycle, due to the relatively low maximum cycle temperature, high thermal efficiency and compactness. Chiefly due to high density and low specific volume of the fluid in the vicinity of the critical point, turbomachines and heat exchangers are compact compared to other Brayton cycles with different operating conditions [1]. According to a study by Angelino [2], the compressor operating condition near the critical point would improve the compressor performance and consequently cycle efficiency. Despite the enormous benefits of designing compressor in the proximity of the critical point, abrupt behavior of the thermophysical properties makes the design and simulation complicated. Figure 1 depicts the isobaric specific heat capacity variations near the critical point. Quite a few researchers have addressed the turbomachinery simulation and design challenges in the vicinity of the critical point [3]–[10]. One of the most used turbomachinery sizing methods is using the specific speed and specific diameter, $n_s - d_s$, diagram proposed by Balje [11]. However,
the predicted performance using the $n_s - d_s$ diagram in the supercritical region is imprecise due to real gas approximation and inconsistent behavior of thermodynamics properties [12]; consequently, a meticulous approach is needed to predict compressor performance through the one-dimensional analysis and computational fluid dynamic (CFD).

Figure 1. Isobaric specific heat capacity variation near the critical point.

Many researchers have developed loss models for turbomachinery design such as Conrad et al. [13], Coppage et al. [14], Jansen [15], Aungier [16] and Rodgers [17]. Implementing the individual enthalpy loss models into the one dimensional design code was evaluated by Lee et al. [12] in the supercritical region; It was concluded that the future development of loss models are crucial to achieve more trustable designs.

In this paper, the authors have attempted to design and simulate the centrifugal compressor based on the individual enthalpy loss models. Due to the small size of the compressor, because of high density near the critical point, skin friction loss is found to play a noticeable role among the internal enthalpy loss models. To shed more light on this matter, the different skin friction loss models are compared and a general correlation for the skin friction loss and the skin friction factor are derived. Validation and verification have been carried out against the experimental measurements from the Sandia laboratory and time-dependent CFD simulations.

2. METHODOLOGY

The most practical and acceptable accuracy set of enthalpy loss models collected by Oh et al. [18] has been implemented in the in-house mean line code (AlFa CCD [19]) to design and evaluate centrifugal compressor performance. The studied set of loss models were evaluated in the supercritical applications previously [12] [18] [20].
Although, in all studied cases based on the loss models, skin friction factor has been assumed constant, or it has been calculated based on the pipe flow correlations. In the AlFa CCD code, the authors attempt to focus on the friction loss and derive a general correlation with acceptable accuracy for the near critical point applications.

Fluid properties have been derived from the NIST Reference Fluid Thermodynamic and Transport Properties Database 9.1 (REFPROP) [21] based on the Span and Wagner equation of state (SW EOS) model [22]. SW EOS covers the CO\textsubscript{2} thermophysical properties from the triple point up to 1100 K and 800 MPa for temperature and pressure, respectively. This model has been employed and validated by considerable number of researchers in the near critical point applications and is been known as the most accurate EOS especially for CO\textsubscript{2} in the supercritical region [8], [23]–[31].

2.1. MEAN LINE DESIGN

The inlet stagnation conditions, the mass flow rate, the rotor rotational speed and the inlet flow angle are the input variables. The velocity triangle definitions used in the study are illustrated in the figure 2.

![Velocity triangle definitions at the leading (left) and trailing (right) edges.](image)

Figure 2. Velocity triangle definitions at the leading (left) and trailing (right) edges.

The centrifugal compressor design is based on the two well-founded equations, Euler and continuity, as follows:

1. \[ h_{t2} - h_{t2} = U_2 C_{w2} - U_1 C_{w2} \]
2. \[ \dot{m} = \rho A C_{a1}. \]

Where subscripts 1 and 2 stand for the inlet and outlet of the impeller, respectively. In the design code, velocity triangles are calculated at three different radiiuses: hub, mean line and the shroud. Mean line radius is estimated as follows

3. \[ r_m = \sqrt{0.5(r_s^2 + r_h^2)} \]
After calculating the velocity triangles at the mentioned locations by using the fluid properties derived from REFPROP, enthalpy loss models are estimated to update the compressor performance parameters.

The set of studied loss models can be classified into internal and external (parasitic) losses. Internal losses consist of incidence, blade loading, skin friction, tip clearance and mixing losses. After updating the compressor performance by considering the internal loss models, external loss models are calculated to take account the extra work of the rotor rotation. External losses are the leakage, the recirculation and the disc friction losses. Enthalpy loss models collected by Oh et al. [18] are summarized in table 1.

Table 1. Individual enthalpy loss models collected by Oh et al. [18].

<table>
<thead>
<tr>
<th>Loss model</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>Incidence loss</td>
<td>Conrad et al. [13]</td>
</tr>
<tr>
<td>Blade loading loss</td>
<td>Coppage et al. [14]</td>
</tr>
<tr>
<td>Skin friction loss</td>
<td>Jansen [15]</td>
</tr>
<tr>
<td>Tip clearance loss</td>
<td>Jansen [15]</td>
</tr>
<tr>
<td>Mixing loss</td>
<td>Johnston and Dean [32]</td>
</tr>
<tr>
<td>Leakage loss</td>
<td>Aungier [16]</td>
</tr>
<tr>
<td>Recirculation loss</td>
<td>Oh et al. [18]</td>
</tr>
<tr>
<td>Disc friction loss</td>
<td>Daily and Nece [33]</td>
</tr>
</tbody>
</table>

Details about the individual loss models can be found in the literature. In this study, only the details of skin friction loss and skin friction coefficient are investigating extensively. The skin friction loss occurs due to the viscous shear forces in the boundary layers at walls inside the impeller. The model proposed by Jansen [15] is defined as

\[
\Delta h_{SF} = 2c_f \frac{L_b}{d_{hb}} \overline{W}^2,
\]

\[
\overline{W} = \left( \frac{2w_2 + w_1 + w_{1h}}{4} \right)
\]

where \( \overline{W} \) stands for the mean relative velocity through the passage, \( d_{hb} \) is the average hydraulic diameter of the blade passage and \( C_f \) is the skin friction coefficient. The flow path length, \( L_b \) is estimated as

\[
L_b = \frac{\pi}{8} \left[ d_2 - \frac{d_{1t} + d_{1h}}{2} - b_2 + 2L_z \right] \left( \frac{4}{\cos \beta_{1t} + \cos \beta_{1h} + 2 \cos \beta_z} \right)
\]

and the axial length of the rotor, \( L_z \) is estimated by the modeled proposed by Aungier [16] as
Where $\phi$ stands for flow coefficient. In order to calculate the skin friction factor, $C_f$, Jansen [15] proposed a correlation which is based on the pipe flow friction calculation as follows

$$c_f = 0.0412 (Re)^{-0.1925}$$

$$Re = \frac{\bar{\rho} \bar{W} d_{hb}}{\mu}$$

and the averaged hydraulic diameter of the rotor blade passage $d_{hb}$ as follows

$$d_{hb} = d_2 \left[ \frac{\cos \beta_2}{\pi + d_2 \cos \beta_2} + \frac{0.5 (d_{s1} + d_{h1})}{\pi + d_{s1} - d_{h1}} \left( \frac{\cos \beta_{s1} + \cos \beta_{h1}}{2} \right) \right].$$

Calculating the skin friction factor based on the pipe flow approximations may underestimate the actual value of friction loss due to curved shape of the blade passage. As suggested by Jansen [15], an average value of 0.006 results in a good agreement with the experimental data for the air compressors. However, due to the abrupt behavior of the viscosity and density in the vicinity of the critical point, employing this averaged value should be examined scrupulously.

Another well-established model for the skin friction factor in the turbulence flows was introduced by Schlichting [34] as follows

$$\frac{1}{\sqrt{4 c_{f_r}}} = -2 \log \left[ \frac{e}{3.71 d} \right]$$

$$\frac{1}{\sqrt{4 c_{f_s}}} = -2 \log \left[ \frac{2.51}{Re_{hb} \sqrt{4 c_{f_s}}} \right]$$

Where $e$ stands for the peak to valley surface roughness. $c_{f_s}$ and $c_{f_r}$ stand for the skin friction factors for fully smooth and rough surfaces, respectively. Centrifugal compressors operate in a wide range of the operating conditions, therefore a general statement for the skin friction loss and the skin friction factor is recommended to cover the laminar and turbulent flows as well as the influence of the surface finish. A weighted averaged model introduced by Aungier [16] can be used when the surface roughness becomes significant

$$Re_e = (Re - 2000)e/d > 60$$

Hence, the turbulent skin friction coefficient is defined as
\[(13) \quad c_f = c_{f_s} + (c_{f_r} - c_{f_s})(1 - 60/Re_e)\]

For a simple annular passage, hydraulic diameter can be assumed as the passage width, but for applying the generalized skin friction model to the compressor passage, the hydraulic diameter of the blade passage proposed by Jansen (equation 9) has been implemented into the weighted averaged model. In order to validate and examine the skin friction coefficient models, experimental data and unsteady CFD simulation over a wide range of operating conditions are needed.

### 2.2. NUMERICAL METHODS

To the best knowledge of the authors of this article, the only open access experimental data of a sCO\textsubscript{2} centrifugal compressor can be derived from the Sandia National laboratories reports [35], [36]. The studied case is the main centrifugal compressor in the Sandia split-flow re-compression sCO\textsubscript{2} Brayton cycle. The unshrouded impeller includes six main and six splitter blades, and the diffuser employs 17 wedge-shaped vanes. Main compressor dimensions are summarized in the table 2.

<table>
<thead>
<tr>
<th>Table 2. Main compressor design dimensions [36].</th>
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<tbody>
<tr>
<td>Impeller diameter ratio (d_2/d_{1h})</td>
</tr>
<tr>
<td>Impeller tip diameter</td>
</tr>
<tr>
<td>Exit blade height</td>
</tr>
<tr>
<td>Blade tip angle (minus is backsweped)</td>
</tr>
<tr>
<td>Blade thickness</td>
</tr>
<tr>
<td>Inlet blade angle at tip</td>
</tr>
<tr>
<td>Normal tip clearance (constant)</td>
</tr>
<tr>
<td>Exit vaned diffuser angle</td>
</tr>
</tbody>
</table>

The mesh dependency test was previously done and by authors [23] and the total amount of around 1.5 million cells found to be sufficient since the compressor performance remained constant by increasing the number of cells. The sufficient fine cells near the walls were generated to ensure the values of \(y^+\) close to unity. Figure 3 shows the Sandia centrifugal compressor geometry and structured mesh. The volute has not been modeled due to lack of geometrical data.

URANS equations were closed through the two equation \(k-\omega\) SST turbulence model of Menter [37]. Convergence criteria of the CFD simulations were based on reduction of Root Mean Square (RMS) momentum, mass and energy residuals below \(10^{-3}\%\), reduction and stability of the imbalances of (difference between inlet and outlet in each zone) mass flow rate, energy and momentum below \(10^{-2}\%\), and the stability in the stage isentropic efficiency. Transient sliding mesh Blade Row interface employing the Fourier
Transformation was defined between the impeller and the vaned diffuser to capture the losses occurring in the transient situation as the flow is mixed between the rotating and stationary zones.

![Figure 3. Geometry and mesh of the Sandia centrifugal compressor.](image)

An external real gas properties (RGP) look-up table has been coupled with the flow solver. RGP table resolution is gradually increased by getting closer to the critical density curve and sufficiency wide range has been used to prevent clipping or extrapolating methods by the flow solver during the simulations. RGP dependency tests have been done by the present authors near the critical point and the optimum resolution of the table was also examined [23], [38]. Boundary conditions at the inlet are defined as total pressure and total temperature, reduced values (normalized with the critical value) are 1.042 and 1.006 for pressure and temperature, respectively. The static back pressure was set as the outlet boundary condition (reduced values are from 1.16 to 1.33 with interval size of 0.2 MPa). Skin friction coefficient can be calculated from the URANS simulations and is formulated as

\[ c_f = \frac{\tau_w}{2\rho_\infty U_\infty^2} \]  

where \( \tau_w \) is the wall shear stress and \( U_\infty \) is the free-stream velocity. In the complicated geometries like inside the compressor flow passage, defining the free-stream velocity is not straightforward. A method introduced by Tiainen et al. [39] estimates the boundary layer thickness as a distance between the impeller blade and the location where the stream velocity is 99.5% of the adjacent point velocity as follows

\[ \frac{du}{dn} = 0.005 \]  

\[ U_{n-1} = 0.995U_n \]  

\[ U_\infty = U_n \]
where the subscript $n$ denotes normal to the wall. The same method is used in the present study to estimate the boundary layer thickness from hub to shroud direction as well to increase the accuracy of the numerical calculation. After locating the boundary layer thickness from each wall inside the impeller, values are averaged at different meridional distances and consequently the skin friction coefficient is calculated.

RESULTS AND DISCUSSION

Based on the different calculation methods, various values of skin friction loss are found. Figure 4 shows the skin friction coefficient distribution along the meridional distance of the studied impeller at the peak efficiency among the studied off-design points. Skin friction coefficient based on the CFD simulation fluctuates along the impeller passage from inlet to outlet. Normalized distances of 0.2 and 0.6 are the locations of the leading edges of the main and splitter blades, respectively. Higher values of skin friction coefficient can be noticed at the leading edges compared to adjustment points.

![Image of Figure 4](image_url)

Figure 4. Comparison of skin friction coefficient.

To validate the performances of the meanline code and the CFD simulation, figure 5 shows their comparison against measurements for the compressor isentropic efficiency along the off-design points at 50 KRPM rotor speed (using the weighed averaged model of skin friction loss). Although, the CFD simulation overestimates the efficiency which results from neglecting the external loss effects, real gas numerical errors and geometrical deviations acceptable trend of CFD simulation and meanline code can be observed.
The importance of skin friction loss can be highlighted by calculating the individual enthalpy loss models from table 1 at the design point. Figure 6 shows that the skin friction loss plays the most important role among all loss sources with share of more than 50% of the total internal losses.
As it can be seen, the friction loss is not constant along the impeller, and constant values may reduce the design accuracy. The model proposed by Jansen for pipe flows (equation 7) shows biggest difference against the CFD averaged value. While the weighted averaged model by Aungier (equation 13) combined with the hydraulic diameter assumption proposed by Jansen (equation 9) predicts the smallest difference. By implementing the hydraulic diameter value of Jansen in to the model proposed by Aungier, the difference is reduced but still around 81.8 % difference appears. The difference between CFD averaged value and the weighted averaged method is around 1.01 %.

CONCLUSION AND FUTURE WORK

Centrifugal compressor design based on the individual enthalpy loss models in the near critical point applications was investigated in this study. Due to small size of the compressor near the critical point (because of high density and low specific volume of fluid), skin friction loss was found to be the most significant and effective loss inside the rotor. Different models for skin friction coefficient were investigated and the result was compared against the URANS CFD simulation. A weighted averaged method proposed by Aungier by implementing the hydraulic diameter of the impeller passage by Jansen showed the best agreement with the CFD result. Also, by comparing the compressor map at constant rotor speed with the experimental measurement conducted at Sandia national lab, acceptable agreement between the meanline code, CFD simulation and measurement was achieved.

Throughout this process, it was clear that further investigation and improvement of the enthalpy loss models are needed. Also, more validation against experimental measurement should be done in order to confirm the presented method as a general statement. Moreover, compressor operates near the critical point and there is possibility of condensation around the suction side of the blades. Further studies are needed to apply the effect of condensation and its loss on the compressor performance and design.

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REFERENCES


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